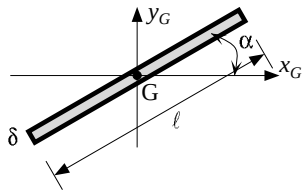
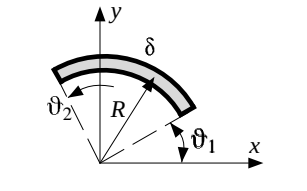
PROPRIETÁ GEOMETRICHE DI ALCUNE AREE PIANE[a]

[a]Autore: Fabrizio Barpi, Gennaio 2006 (http://ulisse.polito.it/matdid/1ing_aer_B4600_T0_0/).

1 SEZIONI SOTTILI ($\delta \ll \ell$)

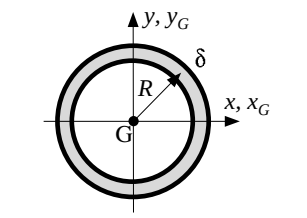


$$A = \delta\ell, \quad x_G = 0, \quad y_G = 0;$$
$$I_{x_Gx_G} = \frac{1}{12}\delta\ell^3 \sin^2\alpha, \quad I_{y_Gy_G} = \frac{1}{12}\delta\ell^3 \cos^2\alpha, \quad I_{x_Gy_G} = \frac{1}{12}\delta\ell^3 \sin\alpha \cos\alpha.$$

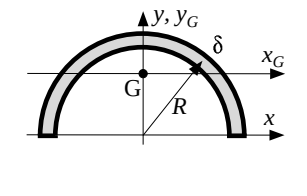


$$A = (\vartheta_2 - \vartheta_1)\delta R, \quad S_x = (-\cos\vartheta_2 + \cos\vartheta_1)\delta R^2, \quad S_y = (\sin\vartheta_2 - \sin\vartheta_1)\delta R^2;$$
$$I_{xx} = \frac{1}{2}(\vartheta_2 - \vartheta_1 - \sin\vartheta_2 \cos\vartheta_2 + \sin\vartheta_1 \cos\vartheta_1)\delta R^3,$$
$$I_{yy} = \frac{1}{2}(\vartheta_2 - \vartheta_1 + \sin\vartheta_2 \cos\vartheta_2 - \sin\vartheta_1 \cos\vartheta_1)\delta R^3,$$
$$I_{xy} = \frac{1}{4}(-\cos(2\vartheta_2) + \cos(2\vartheta_1))\delta R^3.$$

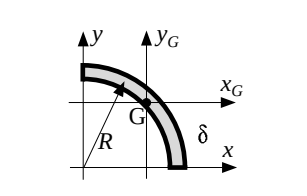
1.1 Casi particolari



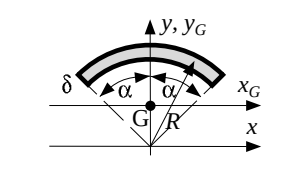
$$A = 2\pi\delta R, \quad x_G = 0, \quad y_G = 0;$$
$$I_{x_Gx_G} = I_{xx} = \pi\delta R^3, \quad I_{y_Gy_G} = I_{yy} = \pi\delta R^3, \quad I_{x_Gy_G} = I_{xy} = 0.$$



$$A = \pi\delta R, \quad x_G = 0, \quad y_G = \frac{2}{\pi}R;$$
$$I_{x_Gx_G} = \left(\frac{\pi}{2} - \frac{4}{\pi}\right)\delta R^3, \quad I_{y_Gy_G} = \frac{\pi}{2}\delta R^3, \quad I_{x_Gy_G} = 0;$$
$$I_{xx} = \frac{\pi}{2}\delta R^3, \quad I_{yy} = \frac{\pi}{2}\delta R^3, \quad I_{xy} = 0.$$

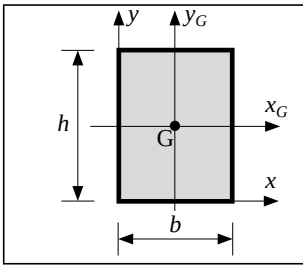


$$A = \frac{\pi}{2}\delta R, \quad x_G = \frac{2}{\pi}R, \quad y_G = \frac{2}{\pi}R;$$
$$I_{x_Gx_G} = \left(\frac{\pi}{4} - \frac{2}{\pi}\right)\delta R^3, \quad I_{y_Gy_G} = \left(\frac{\pi}{4} - \frac{2}{\pi}\right)\delta R^3, \quad I_{x_Gy_G} = -\left(\frac{2}{\pi} - \frac{1}{2}\right)\delta R^3;$$
$$I_{xx} = \frac{\pi}{4}\delta R^3, \quad I_{yy} = \frac{\pi}{4}\delta R^3, \quad I_{xy} = \frac{1}{2}\delta R^3.$$

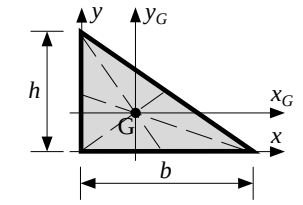


$$A = 2\alpha\delta R, \quad x_G = 0, \quad y_G = \frac{\sin\alpha}{\alpha}R;$$
$$I_{x_Gx_G} = \left(\alpha + \frac{1}{2}\sin(2\alpha) - 2\frac{\sin^2\alpha}{\alpha}\right)\delta R^3, \quad I_{y_Gy_G} = \left(\alpha - \frac{1}{2}\sin(2\alpha)\right)\delta R^3, \quad I_{x_Gy_G} = 0;$$
$$I_{xx} = \left(\alpha + \frac{1}{2}\sin(2\alpha)\right)\delta R^3, \quad I_{yy} = \left(\alpha - \frac{1}{2}\sin(2\alpha)\right)\delta R^3, \quad I_{xy} = 0.$$

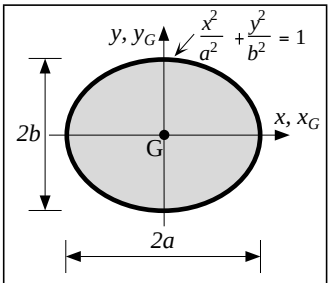
2 SEZIONI COMPATTE



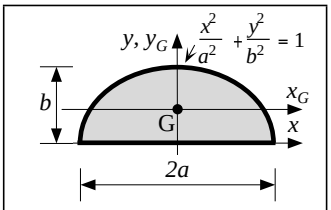
$$A = bh, \quad x_G = \frac{1}{2}b, \quad y_G = \frac{1}{2}h;$$
$$I_{x_G x_G} = \frac{1}{12}bh^3, \quad I_{y_G y_G} = \frac{1}{12}b^3h, \quad I_{x_G y_G} = 0;$$
$$I_{xx} = \frac{1}{3}bh^3, \quad I_{yy} = \frac{1}{3}b^3h, \quad I_{xy} = \frac{1}{4}b^2h^2.$$



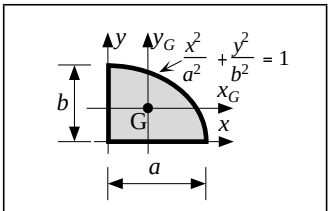
$$A = \frac{1}{2}bh, \quad x_G = \frac{1}{3}b, \quad y_G = \frac{1}{3}h;$$
$$I_{x_G x_G} = \frac{1}{36}bh^3, \quad I_{y_G y_G} = \frac{1}{36}b^3h, \quad I_{x_G y_G} = -\frac{1}{72}b^2h^2;$$
$$I_{xx} = \frac{1}{12}bh^3, \quad I_{yy} = \frac{1}{12}b^3h, \quad I_{xy} = \frac{1}{24}b^2h^2.$$



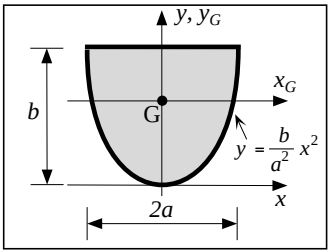
$$A = \pi ab, \quad x_G = 0, \quad y_G = 0;$$
$$I_{x_G x_G} = I_{xx} = \frac{\pi}{4}ab^3, \quad I_{y_G y_G} = I_{yy} = \frac{\pi}{4}a^3b, \quad I_{x_G y_G} = I_{xy} = 0.$$



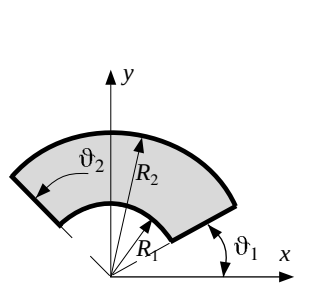
$$A = \frac{\pi}{2}ab, \quad x_G = 0, \quad y_G = \frac{4}{3\pi}b;$$
$$I_{x_G x_G} = \left(\frac{\pi}{8} - \frac{8}{9\pi}\right)ab^3, \quad I_{y_G y_G} = \frac{\pi}{8}a^3b, \quad I_{x_G y_G} = 0;$$
$$I_{xx} = \frac{\pi}{8}ab^3, \quad I_{yy} = \frac{\pi}{8}a^3b, \quad I_{xy} = 0.$$



$$A = \frac{\pi}{4}ab, \quad x_G = \frac{4}{3\pi}a, \quad y_G = \frac{4}{3\pi}b;$$
$$I_{x_G x_G} = \left(\frac{\pi}{16} - \frac{4}{9\pi}\right)ab^3, \quad I_{y_G y_G} = \left(\frac{\pi}{16} - \frac{4}{9\pi}\right)a^3b, \quad I_{x_G y_G} = -\left(\frac{4}{9\pi} - \frac{1}{8}\right)a^2b^2;$$
$$I_{xx} = \frac{\pi}{16}ab^3, \quad I_{yy} = \frac{\pi}{16}a^3b, \quad I_{xy} = \frac{1}{8}a^2b^2.$$

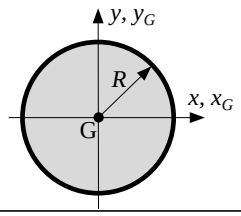


$$A = \frac{4}{3}ab, \quad x_G = 0, \quad y_G = \frac{3}{5}b;$$
$$I_{x_G x_G} = \frac{16}{175}ab^3, \quad I_{y_G y_G} = \frac{4}{15}a^3b, \quad I_{x_G y_G} = 0;$$
$$I_{xx} = \frac{4}{7}ab^3, \quad I_{yy} = \frac{4}{15}a^3b, \quad I_{xy} = 0.$$

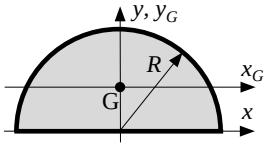


$$A = \frac{1}{2}(R_2^2 - R_1^2)(\vartheta_2 - \vartheta_1);$$
$$S_x = \frac{1}{3}(R_2^3 - R_1^3)(-\cos\vartheta_2 + \cos\vartheta_1), \quad S_y = \frac{1}{3}(R_2^3 - R_1^3)(\sin\vartheta_2 - \sin\vartheta_1);$$
$$I_{xx} = \frac{1}{8}(R_2^4 - R_1^4)(\vartheta_2 - \vartheta_1 - \sin\vartheta_2\,\cos\vartheta_2 + \sin\vartheta_1\,\cos\vartheta_1),$$
$$I_{yy} = \frac{1}{8}(R_2^4 - R_1^4)(\vartheta_2 - \vartheta_1 + \sin\vartheta_2\,\cos\vartheta_2 - \sin\vartheta_1\,\cos\vartheta_1),$$
$$I_{xy} = \frac{1}{16}(R_2^4 - R_1^4)(-\cos(2\vartheta_2) + \cos(2\vartheta_1)).$$

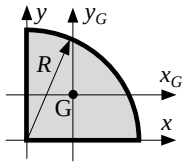
2.1 Casi particolari:



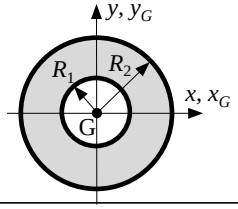
$$A = \pi R^2, \quad x_G = 0, \quad y_G = 0;$$
$$I_{x_G x_G} = I_{xx} = \frac{\pi}{4} R^4, \quad I_{y_G y_G} = I_{yy} = \frac{\pi}{4} R^4, \quad I_{x_G y_G} = I_{xy} = 0.$$



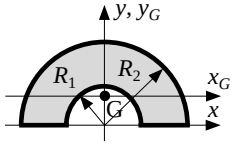
$$A = \frac{\pi}{2} R^2, \quad x_G = 0, \quad y_G = \frac{4}{3\pi} R;$$
$$I_{x_G x_G} = \left(\frac{\pi}{8} - \frac{8}{9\pi} \right) R^4, \quad I_{y_G y_G} = \frac{\pi}{8} R^4, \quad I_{x_G y_G} = 0;$$
$$I_{xx} = \frac{\pi}{8} R^4, \quad I_{yy} = \frac{\pi}{8} R^4, \quad I_{xy} = 0.$$



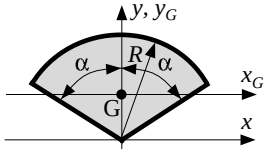
$$A = \frac{\pi}{4} R^2, \quad x_G = \frac{4}{3\pi} R, \quad y_G = \frac{4}{3\pi} R;$$
$$I_{x_G x_G} = \left(\frac{\pi}{16} - \frac{4}{9\pi} \right) R^4, \quad I_{y_G y_G} = \left(\frac{\pi}{16} - \frac{4}{9\pi} \right) R^4, \quad I_{x_G y_G} = -\left(\frac{4}{9\pi} - \frac{1}{8} \right) R^4;$$
$$I_{xx} = \frac{\pi}{16} R^4, \quad I_{yy} = \frac{\pi}{16} R^4, \quad I_{xy} = \frac{1}{8} R^4.$$



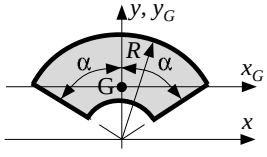
$$A = \pi (R_2^2 - R_1^2), \quad x_G = 0, \quad y_G = 0;$$
$$I_{x_G x_G} = I_{xx} = \frac{\pi}{4} (R_2^4 - R_1^4), \quad I_{y_G y_G} = I_{yy} = \frac{\pi}{4} (R_2^4 - R_1^4), \quad I_{x_G y_G} = I_{xy} = 0.$$



$$A = \frac{\pi}{2} (R_2^2 - R_1^2), \quad x_G = 0, \quad y_G = \frac{4}{3\pi} \frac{R_1^2 + R_1 R_2 + R_2^2}{R_1 + R_2};$$
$$I_{x_G x_G} = \frac{\pi}{8} (R_2^4 - R_1^4) - \frac{8}{9\pi} \frac{(R_2^3 - R_1^3)^2}{R_2^2 - R_1^2}, \quad I_{y_G y_G} = \frac{\pi}{8} (R_2^4 - R_1^4), \quad I_{x_G y_G} = 0;$$
$$I_{xx} = \frac{\pi}{8} (R_2^4 - R_1^4), \quad I_{yy} = \frac{\pi}{8} (R_2^4 - R_1^4), \quad I_{xy} = 0.$$



$$A = \alpha R^2, \quad x_G = 0, \quad y_G = \frac{2}{3} \frac{\sin \alpha}{\alpha} R;$$
$$I_{x_G x_G} = \frac{1}{4} \left(\alpha + \frac{1}{2} \sin(2\alpha) - \frac{16}{9} \frac{\sin^2 \alpha}{\alpha} \right) R^4, \; I_{y_G y_G} = \frac{1}{4} \left(\alpha - \frac{1}{2} \sin(2\alpha) \right) R^4, \; I_{x_G y_G} = 0.$$
$$I_{xx} = \frac{1}{4} \left(\alpha + \frac{1}{2} \sin(2\alpha) \right) R^4, \quad I_{yy} = \frac{1}{4} \left(\alpha - \frac{1}{2} \sin(2\alpha) \right) R^4, \quad I_{xy} = 0.$$



$$A = (R_2^2 - R_1^2)\alpha, \quad x_G = 0, \quad y_G = \frac{2}{3} \frac{\sin \alpha}{\alpha} \frac{R_1^2 + R_1 R_2 + R_2^2}{R_1 + R_2};$$
$$I_{x_G x_G} = \frac{1}{4} (R_2^4 - R_1^4) \left(\alpha + \frac{1}{2} \sin(2\alpha) \right) - \frac{4}{9} \frac{(R_2^3 - R_1^3)^2}{R_2^2 - R_1^2} \frac{\sin^2 \alpha}{\alpha},$$
$$I_{y_G y_G} = \frac{1}{4} (R_2^4 - R_1^4) \left(\alpha - \frac{1}{2} \sin(2\alpha) \right), \quad I_{x_G y_G} = 0;$$
$$I_{xx} = \frac{1}{4} (R_2^4 - R_1^4) \left(\alpha + \frac{1}{2} \sin(2\alpha) \right), \quad I_{yy} = \frac{1}{4} (R_2^4 - R_1^4) \left(\alpha - \frac{1}{2} \sin(2\alpha) \right), \quad I_{xy} = 0.$$

NOTE